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The Effect of Tire Tread in Parametric Analyses of Tire-Soil Systems

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THE EFFECT OF TIRE TREAD IN PARAMETRIC ANALYSES OF TIRE-SOIL SYSTEMS

M.G. Bekker

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ABSTRACT

The paper is, in a sense, a sequel to the Report published by the National Research Council of Canada, in 1983, and to the paper published by the Society of Automotive Engineers, in 1974.

It reports corrections, refinements, validation, and extension of bevameter methods in parametric analyses of pneumatic tire-soils systems.

Considerations of detailed dimensions, form, internal structure, configuration (duals, singles), tread design, inflation pressure, load, and cost of tires are discussed in the realm of off-road machinery.

Predictability of vehicle performances is illustrated in various soils by numerical examples and test results. Mineral and organic soils and snow are considered.

The work has been based, to a large extent, on recent studies performed by the author for the Canadian and Italian National Research Councils.

RÉSUMÉ

Le présent document est, dans un certain sens, une suite aux documents publié par le Conseil national de recherches du Canada en 1983, et par la Society of Automotive Engineers en 1974.

Il donne un aperçu des modifications, des perfectionnements, de la validation et des développements des méthodes bevamètres dans les analyses paramétriques des systèmes d'adhérence pneumatique aux divers types de sol.

On y traite de question comme les dimensions, la forme, la structure interne, la configuration (doubles, simples), la conception de la bande de roulement, la pression de gonflage, la charge et le coût des pneus dans le domaine des véhicules tout-terrain.

On établit les prévisions quant au rendement du véhicule sur divers type de sols par des exemples numériques et les résultats de tests. On a prévu des essais sur un sol minéral ou organique et dans la neige.

Ce travail est fondé en grande partie sur des études récentes réalisées par l'auteur à l'intention des conseils de recherches nationaux italien et canadien.

THE EFFECT OF TIRE TREAD IN PARAMETRIC ANALYSES OF TIRE-SOIL SYSTEMS

M.G. Bekker

PARAMETRIC ANALYSIS

The qualitative approach to tire performance evaluation seen even in professional literature as illustrated by Fig. 4 of ref. 1 often yields opinions, instead of facts.

Parametric analyses, which have been used in other engineering branches for years, alleviate the problem by enabling one to quantitatively predict design-performance parameters of specific terrain-vehicle systems in terms of figures and statistical inference (2, 3). This paper reviews and updates the parametric analyses as applied to pneumatic tires and introduces the consideration of tread effect. It is based on my most recent work published by the Canadian and Italian National Research Councils (4, 5), which is suggested for background studies.

HISTORY AND THE NATURE OF PROGRESS

The first known parametric model of the rigid wheel on soft ground is due to Bernstein (1913) and the Russian engineer, Goriatchkin (1936). Credit must also be given to Morin (1865) for his empirical graphic correlation of wheel dimensions, load and motion resistance (6). An elastic wheel on yielding ground was the subject of a variety of treatments, mainly empirical or exceedingly theoretical.

General Parametric Modeling

General parametric modeling of pneumatic tires which behave like elastic, or rigid wheels, depending on soil consistency, was first worked out at the U.S. Army's Land Locomotion Laboratory in Detroit, Michigan, in the late 1950's. The results were published in 1960 (7). It is this work, which I have continuously developed and refined until recently, that is the basis of the present paper.

The 'longevity' of the 1960 tire model appears to be due to the fact that it is the only existing model based on a generalized soil value system applicable to any ground surface, mineral or organic, or snow, having any configuration in depth, homogeneous, or stratified. The soil value system used here is derived from experimental records of ground-load deformation which simulates vehicle action upon the ground. Pertinent measurements are performed with the instrument called a bevameter. The data thus obtained yield soil values compatible with applied mechanics, and enable one to

construe equations with soil and vehicle parameters which can be mathematically optimized. Details and procedures are described in refs. 3 to 7, and in other publications, in various languages.

Figures 1 and 2 show examples of soil deformation functions in shear and penetration, in a stubble and ploughed field measured by Söhne and Sonnen by means of a bevameter (8). For the convenience of the reader the data processing was reconstructed 'by hand' as shown in ref. 7. The measurements were not statistically significant. In order to obtain a useful approximation, each set of four measurements was enclosed by an envelope (shaded area) and its upper and lower boundaries were used for data band processing (3).

The 1960 Tire Model

The 1960 tire model is based on the assumption of an elastic, pliable torus envelope which deflects with a geometrical precision. The spuds, lugs, and grousers are not considered as such. Their effect is inherent in tire load-inflation-deflection characteristics input supplied by the manufacturer, in the form of carcass stiffness pressure p_c. Numerous computations show that the parameters predicted by this mode are in fair-to-good agreement with experiment (4, 5), when tire sinkage is large enough in comparison to lug height, so that the lug effect may be neglected. It is then assumed that the inter-lug spaces fill with soil trapped by the tire, which thus behaves like a 'smooth' wheel. However, some soils may be strong enough to support the tire with the tips of the lugs, or with the minimum carcass contact with the ground. This has necessitated the most recent development of another addition to the 1960 model as first presented in the section entitled "Tire Tread and Lug Effect".

Tire Operating Mode

The criterion of tire operating mode (TOM), which denotes if the tire is rigid or elastic under prevailing conditions, also was refined in the 1970's by using the rigid and elastic tire sinkages z_r and z_e, instead of the 'critical' pressure (7). Depending on which sinkage is smaller, the tire is rigid, or elastic.

Carcass Flexing Resistance

The carcass flexing resistance R_t, as proposed in Equation 11, ref. 2, was found to be inaccurate if not incorrect, and has been replaced since 1976 by the well-tested formula reported here, in Equation [6].

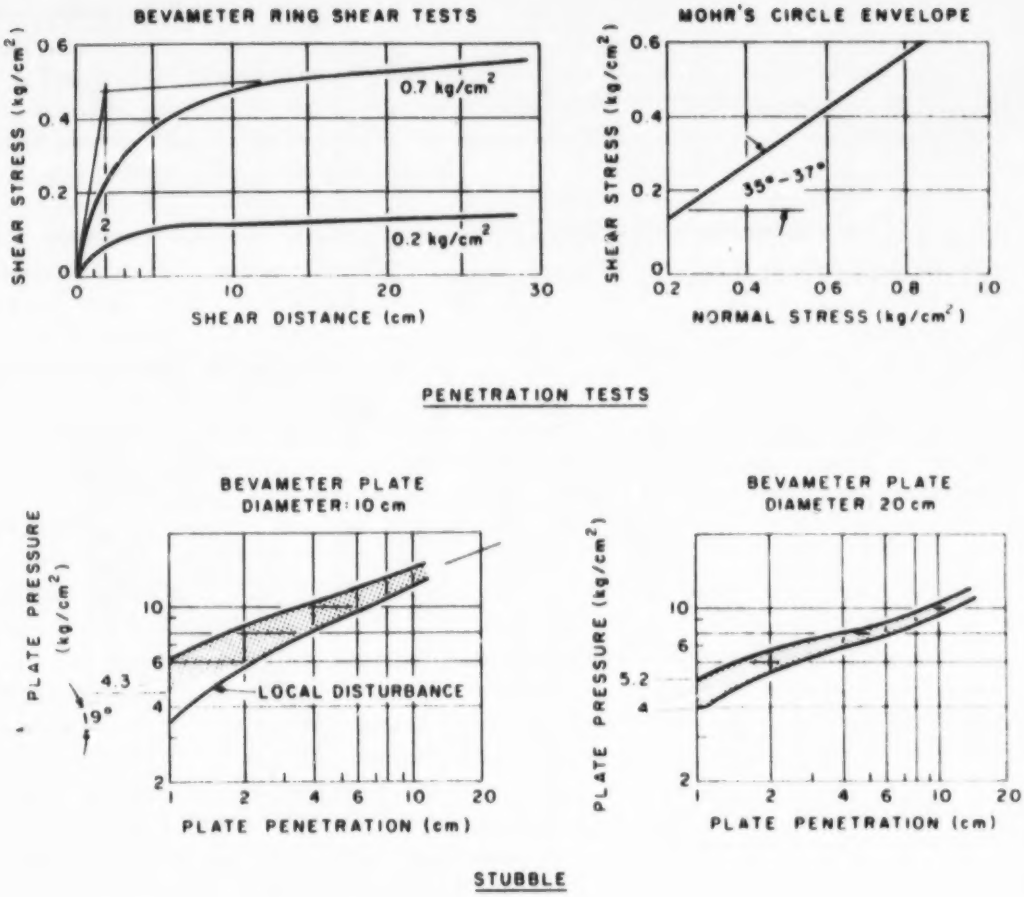

Figure 1. Copies of bevameter measurements made in a ploughed field, and the data processing. Additional comments in the text. (After Söhne and Sonnen (8).)

Soil Parameters Calculation (metric system)

Low Values Envelope:

$k_\phi = (0.9 \times 10 - 1.3 \times 5)/(10 - 5) = 0.5$

$k_c = (1.3 - 0.9)10 \times 5/(10 - 5) = 4.0$

High Values Envelopes:

$k_\phi = (1.3 \times 10 - 1.5 \times 5 \times 5)/(10 - 5) = 1.1$

$k_c = (1.5 - 1.3)\ 10 \times 5/(10 - 5) = 2.0$

$n = \tan 38° = 0.78$

$\phi = 31°$ (see comments in the text)

$c = 0.04$ (estimated by Söhne and Sonnen)

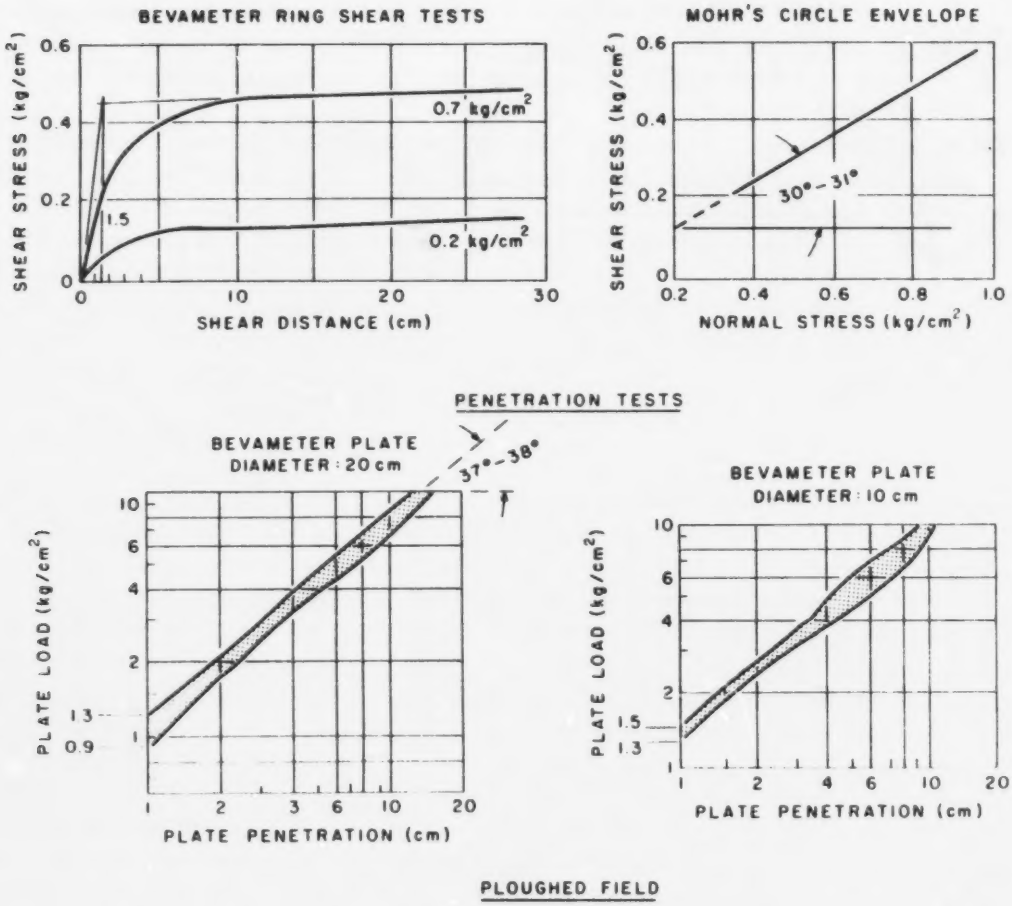

Figure 2. Copies of bevameter measurements made in a stubble covered field, and the data processing. Additional comments in the text. (After Söhne and Sonnen (8).)

Soil Parameters Calculation (metric system)

Low Values Envelope:

$k_\phi = (4 \times 10 - 4.3 \times 5)/(10 - 5) = 3.7$

$k_c = (4.3 - 4)\ 5 \times 10/(10 - 5) = 3.0$

High Value Envelope:

$k_\phi = (5.2 \times 10 - 6 \times 5)/(10 - 5) = 4.4$

$k_c = (6 - 5.2)5 \times 10/(10 - 5) = 8.0$

$n = \tan 19° = 0.34$

$\phi = 37°$ (see comments in the text)

$c = 0.04$ (estimated by Söhne and Sonnen)

Truncation of Taylor Series

The usual truncation of the Taylor Series, which simplified the integration of the Bernstein equation (6) due to consideration of the first two members, was extended to five members, thus providing the necessary accuracy for higher n-values.

However, for the sake of simplicity, Equations [15] and [16] were reported in this paper in the original form which well serves the purpose for $n < 1.5$.

Stratified Soil Evaluation

Stratified soil evaluation was gradually improved since the publication of the original approach in 1973 (9). Details will be found in refs. 4 and 5, and in this paper, in the section concerned with parameter calculations.

Drawbar Pull

The drawbar pull is defined, as usual with the modified Coulombian equation (6, 7), assuming that the working length of the ground contact area is $l_1 + l_2$ and l_0 for small sinkage, and $l = l_1 + 0.3 \times l_2$ for deep sinkage. This takes into account the action of the curved portion of tire print. Numerous computations show that the thrust equation is very sensitive to K and ϕ values. The latter are usually overestimated. Bevameter technique should be improved.

At this time, the existing method (3) produces fair results if properly interpreted. However, the use of rubber plated shear heads as used by Wong (11) is recommended. Furthermore, all bevameter tests should be performed under unconfined soil conditions.

Numerous attempts to correlate the calculations with experiment clearly show the lack of empirical data, or the inadequacy of their measurement in a meaningful way. The lack of coordination of bevameter and field tests in the same time-space framework is the main deficiency, besides the lack of standardization of tests. Frequent criticism of the theory was based on poor field measurements rather than on theory's error.

Computerized Calculations

Computerized calculations have greatly facilitated the work described, even though I have used as simple a tool as the programmable HP 41C. The programs thus far developed include:

regular elastic tire
wide tire (terratire type)
rigid tire
tire with flat tread

The tire tread and lug effect has not yet been programmed, and the Tables of the Appendix related to that effect contain 'by hand' calculations.

MATHEMATICS OF TIRE CALCULATIONS

For the convenience of the reader, all the main equations and the sequence of procedures in tire parametric analyses are referred to as shown below.

Tire on Rigid Road

A tire on a rigid road (Fig. 3f) deflects in a way similar to the tire on a yielding surface (Fig. 3c and Fig. 4), unless it is rigid (Fig. 3a). Using denotations shown on the list included and on the drawings, calculate δ:

[1] $$(2r\delta - \delta^2)(D\delta - \delta^2) = (W/4\lambda p_g)^2$$

Calculate b_0 if $b_0 < b_{tr}$:

[2] $$b_0 = 2(2r\delta - \delta^2)^{1/2}$$

For $b_0 > b_{tr}$ use:

[3] $$\delta = \{D - [D^2 - 4(W/2\lambda b_{tr} p_g)^2]^{1/2}\}/2$$

Calculate,

[4] $$\alpha = \cos^{-1} [(D - 2\delta)/D]$$

Calculate,

[5] $$\varepsilon = 1 - \exp(-k\delta/h)$$

where k = 15 for regular and k = 7 for radial tires. Calculate the tire rolling resistance due to carcass flexing,

[6] $$R_t = [3.581 b_0 D^2 p_g \, \varepsilon(0.0349\alpha - \sin 2\alpha)]/\alpha(D - 2\delta)$$

For numerical examples and their verification see refs. 4 and 5.

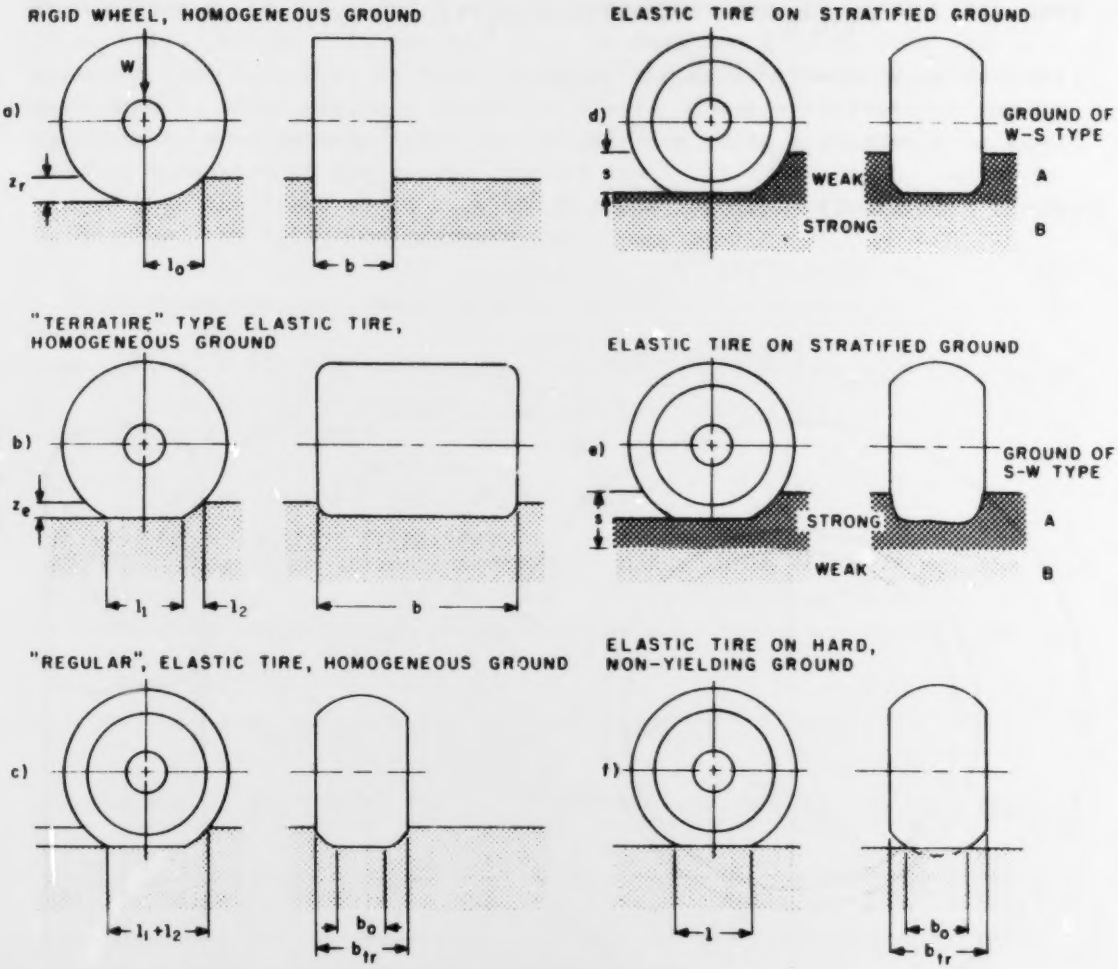

Figure 3. Geometry and some parameters of basic soil-tire systems

Tire on Yielding Homogeneous Ground

A tire on yielding homogeneous ground (Figs 3b, 3c and Fig. 2) is evaluated by means of equations which have to be solved by iteration, assuming the initial value of b_0,

[7] $$F = 1/(n + 1), \quad K = F/(n + 2), \quad B = WF/2$$

[8] $$A = b_0 p_g[(F^2/2) - K]$$

[9] $$z_e = \{p_g/[(k_c/b_0) + k_\phi]\}^{1/n}$$

[10] $C = [b_0 p_g z_e/(n + 1)][(D/2) - (z_e/3)]$

[11] $l_2 = [B - (B^2 - 4AC)^{1/2}]/2A$

[12] $l_1 = (W/b_0 p_g) - l_2 F$

[13] $\delta = (D/2) - [(D/2)^2 - (l_1/2)^2]^{1/2}$

[14] $b_0 = 2(2r\delta - \delta^2)^{1/2}$

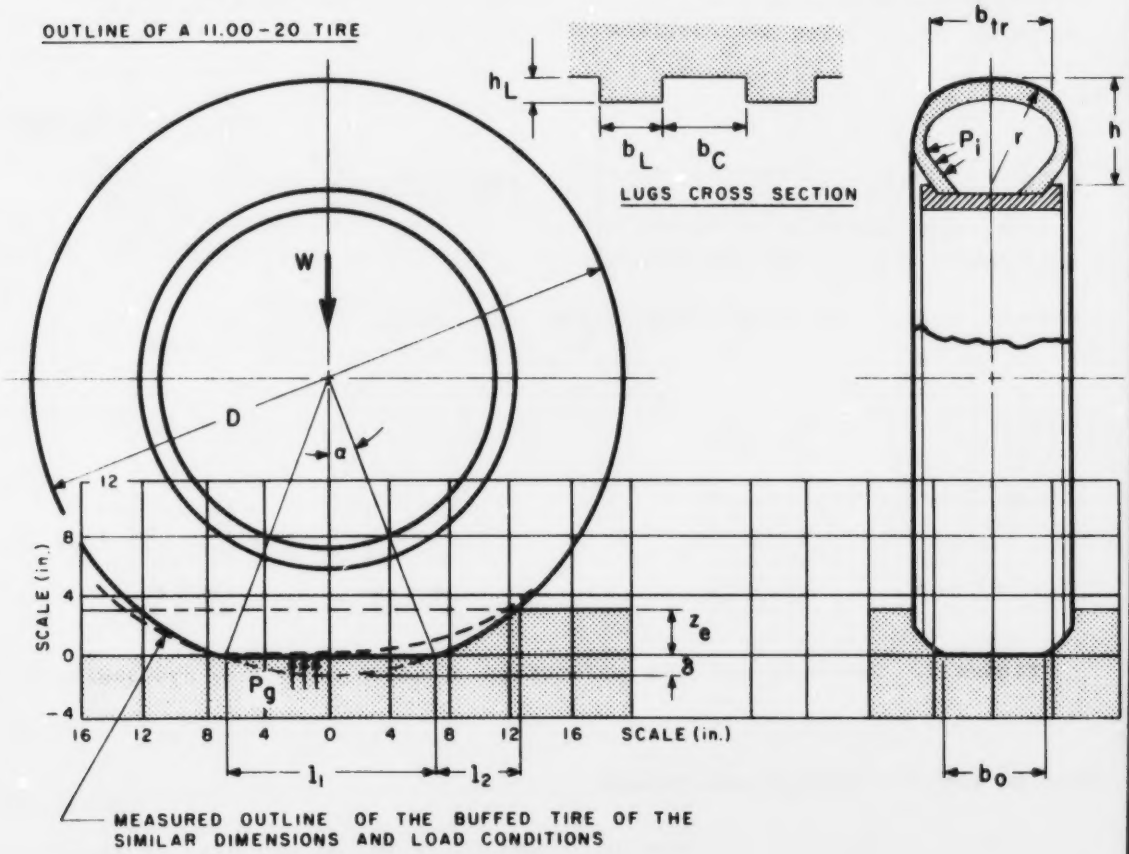

Figure 4. Outline of a regular tire and its basic dimensions used in the mathematical model of 1960. The tire motion resistance, sinkage and deflection silhouette were computed for a buffed 11.00-20 tire as shown in Table 1A. The agreement between computed and measured parameters is very good as shown by tire deflection, sinkage and motion resistance. Details in the text.

Then values z_e, l_1, l_2, and δ are given by Equations [9], [11], [12], [13], if $b_0 < l_1$. For $b_0 > l_1$ use $b_{tr} = b_0$, in Equation [8]; $b_0 = l_1$, in Equation [9]; $b_0 = b_{tr}$, in Equations [10] and [12]. Then, by iteration, determine l_1 which yields δ (Equation [13]). These substitutions may be remembered, when assuming that the denominator of k_c must always be the smaller dimension of the ground contact area. In addition, b_0 cannot be larger than the tread width b_{tr}, which comes into the picture when the load is high and the inflation pressure low.

To check TOM, calculate z_r (Fig. 3a).

[15] $$z_r = \{3W/b(3 - n)[(k_c/b) + k_\phi)]D^{1/2}\}^{2/(2n+1)}$$

if $b < l_0$. If not, use

[16] $$z_r = \left(3W/b(3 - n)\{[k_c/(Dz - z_r^2)^{1/2}] + k_\phi\}D^{1/2}\right)^{2/(2n+1)}$$

Upon computing these values compare z_e and z_r. Then for $z_e > z_r$, the tire is RIGID; for $z_e < z_r$, it is ELASTIC. Tire motion resistance due to vertical soil compaction is calculated for both rigid and elastic TOM by means of equation,

[17] $$R_c = b_{0,\text{or } tr}[(k_c/b_{0,\text{or } l_1}) + k_\phi]z_{e,\text{or } r}^{n+1} \quad /(n+1)$$

and the motion resistance due to horizontal soil deformation (bulldozing) is

[18] $$R_b = b_0(0.6667z_{e,\text{or } r}cK_c + 0.5z_{e,\text{or } r}^2\gamma K_\gamma)$$

The sum of these resistances yields the total,

[19] $$R = R_c + R_t + R_b$$

Note, that sinkage is limited by the equation,

[20] $$z_{e,\text{or } r} < (D/2)(1 - \cos\mu)$$

In general, sinkage greater than 30% of the wheel radius immobilizes the vehicle, and has only an academic significance.

The value of μ usually equals the co-efficient of soil adhesion to the tire. Equations [15] and [16] are based on the two-member truncated Taylor series as discussed before.

Tire on Yielding Non-homogeneous Ground (Figs. 3d and e)

Computations of laboratory and field cases indicate that a two-layer substitute system is most often sufficient to determine tire performance. This simplifies the solution first considered in ref. 9 and later in refs. 4 and 5.

Accordingly, for a two-layer soil of W-S type (Fig. 3d), first calculate load W_A supported by the top layer. Assume rigid TOM:

[21] $$W_A = 0.3333b_{tr}[(k_{cA}/b_{tr}) + k_\phi](3 - n_A)D^{1/2}s^{(2n_A+1)/2}$$

Next, assuming that the new tire load acting on layer B is $W - W_A$, proceed with the solution of Equations [7]-[16]. This leads to the solution of motion resistances, Equations [6], [17]-[19].

However, in the case considered, additional motion resistance due to the tire cutting through A-layer, must be added.

[22] $$R_{cA} = b_{tr}[(k_{cA}/b_{tr}) + k_\phi]s^{n_A+1)}/(n_A + 1)$$

[23] $$R_{cA} = b_{tr}(0.3333csK_c + 0.5s^2\gamma K_\gamma)$$

Note that in a compactible snow, the rear wheels of the vehicle usually roll on a strong icy layer and produce only tire carcass flexing resistance R_t to be determined from Equation [6]. In such a case R_{cA} and R_{cA} may often be neglected.

Soils of S-W type (Fig 3e) are calculated only for the top layer A, using the same method as for homogeneous soils, upon checking the puncture pressure (3). The bottom line for tire performance is defined by that pressure, when layer B cannot support an appreciable load.

TIRE TREAD AND LUG EFFECT

Tire tread with lugs which are spaced at average distances smaller than their ground contact width (smaller dimension) do not require special consideration. They may be handled, generally speaking, with the 1960 model. More experimental work and computation are needed to pinpoint the side effects of the multiplicity of tread designs.

Chevron Tread and Ribs Parallel to Wheel Plane

Chevron tread and ribs parallel to wheel plane were thus far considered, respectively. Figures 5 and 6, drawn to approximate scale, show that in a stubble (left sides of the drawings) only the lugs support tire load, while in a ploughed field (right side), both the lugs and the carcass are in ground contact. The soil was defined in bevameter values as shown in Figs. 1 and 2.

The bearing area of Chevron lugs may be measured directly, or defined in terms of a coefficient η_2, which is the percentage of the ground contact area of the carcass A/C occupied by the lugs. The tire area engaged with the ground is

[24] $$A/C = (l_1 + 0.3\ l_2)b_0\eta_1$$

where l_1, l_2, and b_0 are the values previously computed with Equations [7]-[14]. Then, the lug contact area is

[25] $$A/L = (A/C)\eta_2$$

With lug area determined, calculate the sinkage,

[26] $$z_L = \{W/(A/L)[(k_c/b_L) + k_\phi]\}^{1/n}$$

if $z_L < h_L$, the compaction motion resistance is determined from equation,

[27] $$R_c = (A/L)[(k_c/b_L) + k_\phi]z_L^{n+1}/l_1(n + 1)$$

[28] $$R_b = b_o(0.6667cz_LK_c + 0.5z_L^2\gamma K_\gamma)$$

R_t is given by the previously quoted equation [6].

In the case of lug-plus-carcass contact with the ground ($z_L > h_L$), only the "net" carcass contact area, A/CN, carries the remaining load which is not carried by the lugs.

[29] $$A/CN = A/C - A/L$$

Upon calculating Equations [24]-[26], or measuring the areas from the footprint, define the balance of forces by equation,

[30] $$(A/CN)[(k_c/b_c) + k_\phi]z_c^n + (A/L)[(k_c/b_L) + k_\phi](h_L + z_c)^n = W$$

where z_c is carcass sinkage. Other dimensions are shown in Fig. 5.

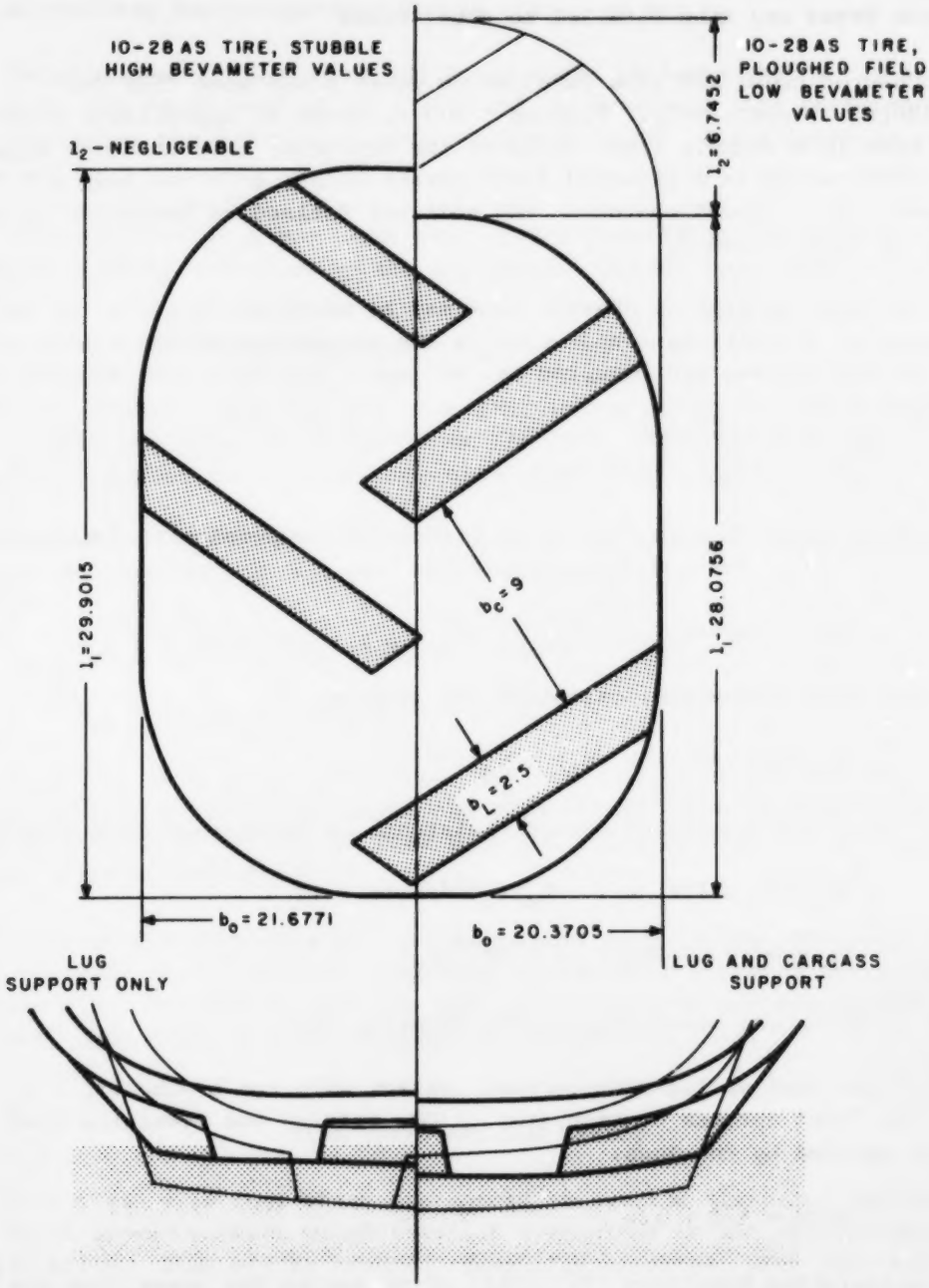

Figure 5. 10-28AS Tire and the basic dimensions of lug pattern. Computation of the footprints shown here are exemplified in Tables A3 and A4. Tire support is provided by the lugs only (left), or by the lugs and the carcass (right).

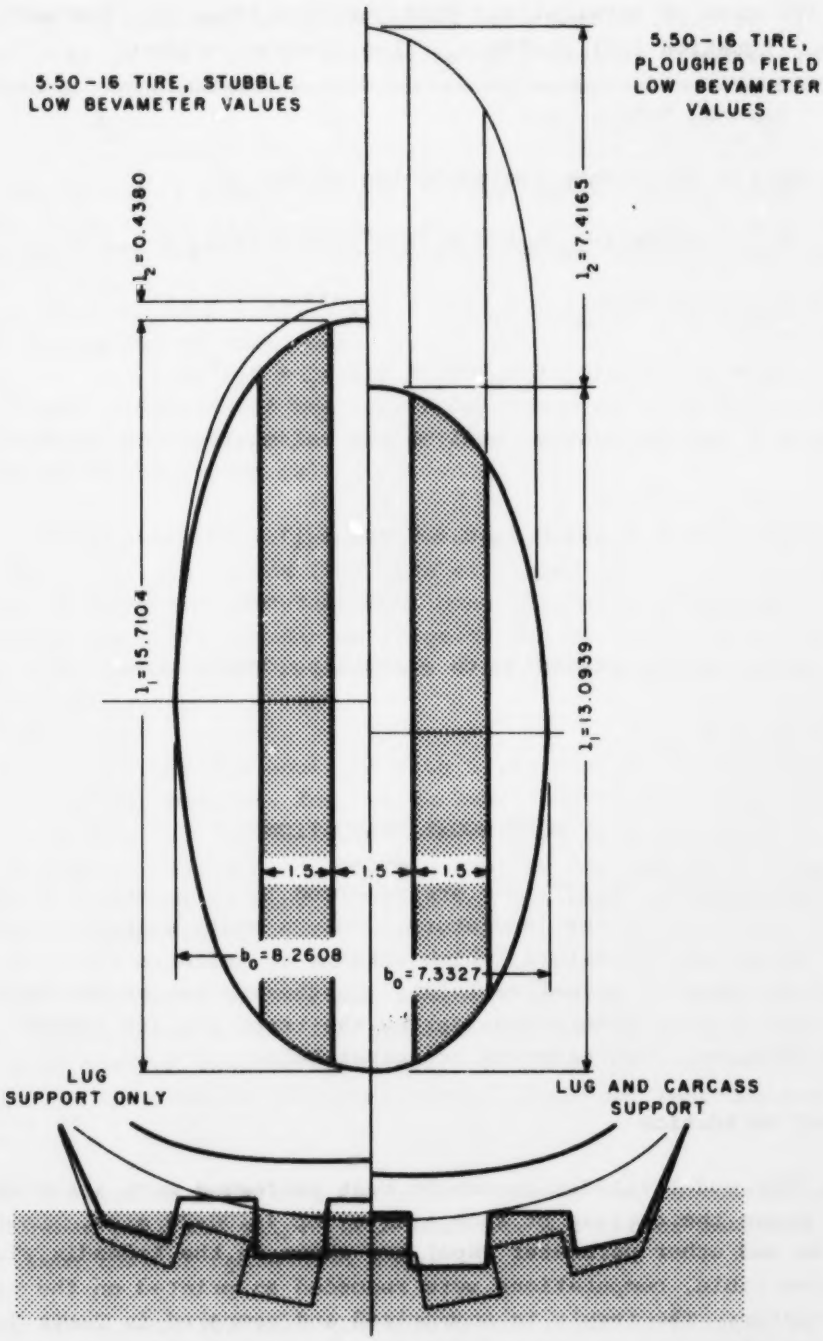

Figure 6. 5.50-16 Tire and the basic dimensions of lug pattern. Computations of the footprint shown here are exemplified in Table A2. Tire support like that in Fig. 5. Greater sinkage, however, makes the lug action less significant from a motion resistance viewpoint.

In the case of parallel rib configuration (Fig. 6), the method remains the same. Equation [30] yields z_c. Lug sinkage is then

$$z_L = z_p + h_L$$

and tire motion resistance including lug action is

[31] $$R_{cC} = (A/CN)[(k_c/b_c) + k_\phi)z_c^{n+1}/(n + 1)(l_1 + 0.3 \times l_2)$$

[32] $$R_{cL}(A/L)[(k_c/b_L) + k_\phi](z_c + h_L)^{n+1}/(n + 1)(l_1 + 0.3 \times l_2)$$

[33] $$R_b = b[0.6667(z_c + h_L)cK_c + 0.5(z_c + h_L)^2\gamma K_\gamma]$$

Soil thrust H and the drawbar pull DP are calculated in all the cases as usual,

[34] $$DP = \left[b(l_1 + 0.3 \times l_2)c + W \tan \phi\right]\left[1 - [K/i_0(l_1 + 0.3 \times l_2)] \times \left\{1 - \exp [-i_0(l_1 + 0.3 \times l_2/)K]\right\}\right]$$

where b is b_0, or b_{tr} depending on prevailing conditions.

$$DP = H - R_c$$

MORE MODEL VALIDATIONS

Extensive model validations are reported in refs. 4 and 5. Tests were primarily performed in the laboratory. Only a small number of tests is known in technical literature, which attempt to validate the tire model in the field by means of actual vehicles. In these attempts the main problem was the lack of tire models considering the tread and lug effect. This has been now remedied. Two examples are given below.

1960 Model Validation

The 1960 model validation may be best performed with a buffed tire. Figure 4 shows the outline of such a tire, of 11.00-20 size. Soil properties and other parameter input are shown in the Appendix, Table A1. In the same table, computations were recorded as printed on the tape of HP 41C calculator. The result is summarized and compared in Table 1. The results are close enough, in all the detail of load and geometry. Note the assumed and measured deflection contours (Fig. 4).

Calculations reported in the Appendix, Table 1A, take approximately 1.5 min, depending on the accuracy of the originally guessed b_0.

Table 1. Calculated and measured values of a buffed 11.00-20 tire, in sand, at 3000 lb load and 15 psi inflation pressure. Controlled test

	z_e	l_2	l_1	δ	R_c	R_b	R_t	R
Calculated	4.4421	8.8920	13.5007	1.0952	401.4	130.2	49.8	581.4
Measured	3.2	6.0	13.0	0.9	-	-	-	615.0

Field Test Validation of Tire Model

Field test validation of the tire model based on 1960 conjecture, and on the consideration of tread and lug effect, is exemplified here, by data measured by Söhne and Sonnen (8).

Since the bevameter soil records were not sampled statistically as it is done today, e.g., in Canada (10), the envelopes of four penetration curves were considered, thus yielding a band of possible solutions, for high (HBV) and low (LBV) boundaries (Figs. 1 and 2). The 'by hand' data processing reported on these Figures has since been replaced by the Canadian computerized method (10).

The tested tractor was equipped with 5.50-16 tires in the front, at 1.5 kg/cm^2 inflation pressure for 272 kg load, and with 10-28AS tires at 0.8 kg/cm^2 pressure and 700 kg load. Other input data are shown in the Appendix, Tables A2, A3, and A4, which exemplify the method of calculating the drawbar pull of the tractor vs. slip for the sake of comparing it with the measured values (Fig. 7).

Note that the angle of friction measured in Figs. 1 and 2 was lowered to 24° for ploughed and to 28° for stubble covered soil. This was done using measured tractor values, which were much smaller than bevameter values. The latter were at fault because of incorrect instrumentation (8).

Figure 7 shows an excellent agreement between the calculated and measured drawbar pull and slip. Note that the scatter of measured values is greater than that of calculated ones.

I have often observed that the theory was rejected because it did not fit the empirical data. But the error was not in the theory. The measurements were incorrect, or misinterpreted.

Numerous calculations show that bevameter measurements and field test verifications must be performed in the same time-space frame. Even then,

the point-to-point correlation between prediction and measurement may be impossible. Only the statistical approach makes sense. It offers enormous gain, when compared with present subjective and rather qualitative predictions. The following examples, based on deterministic consideration, illustrate the point.

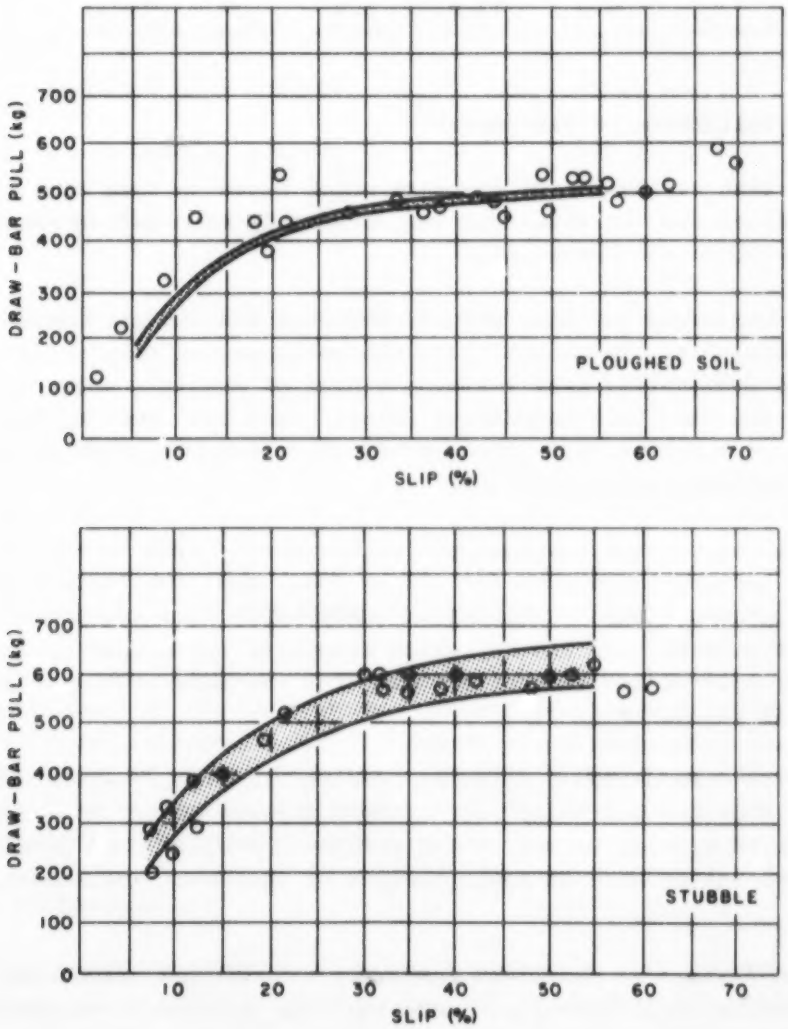

Figure 7. Computed values of drawbar pull for a tractor equipped with 5.50-16 and 10-28AS tires in a ploughed and stubble covered field are shown by shaded areas. Measured values for that tractor, by the circles. Note that the measurements, particularly at low slip, show more scatter than the computations. Measured input after Söhne and Sonnen (8).

EXERCISING TIRE OPTIONS

In this section the drawbar-slip curves are considered. They are an important factor in tractor selection for a given soil because they yield other performance parameters such as travel efficiency, fuel consumption, power, cost, etc.

A hypothetical heavy 4×4 agricultural tractor of basic weight 88 kN is provided with three tire options in single and dual configurations. The load is evenly distributed on both axles at optimum pull and 30-50% slip. Table 2 gives further detail. Figure 8 shows the performance of these tires in a hypothetical clayey soil, for one wheel only. Such one-wheel performance is useful in a first rough estimate of parameters involved. Figure 9 shows performance of some of these tire options in a hypothetical

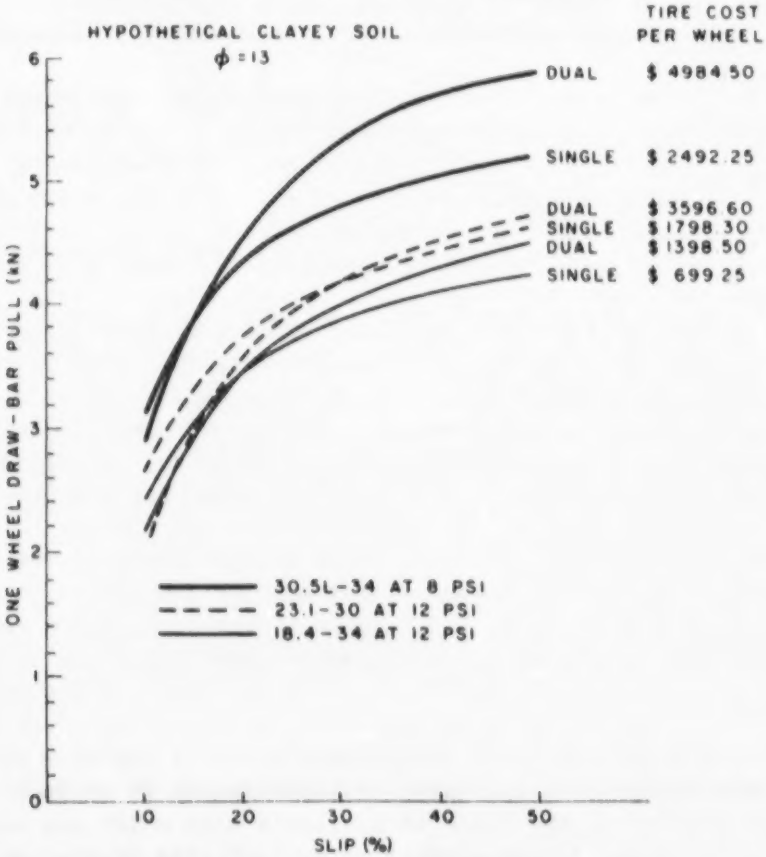

Figure 8. One wheel drawbar pull vs. slip for a set of tire options available with a heavy 4×4 tractor. The hypothetical, though plausible, clayey soil has a low angle of friction $\phi = 13°$. Note the tire cost per wheel in dual and single configurations as opposed to performance.

Table 2. Hypothetical tractor input data and tire cost

Gross vehicle weight (kN)	Tire option	Inflation Pressure (psi)	Config.	Tire cost ($)
88.00	18.4-34	20	Single	2797
92.96	18.4-34	12	Dual	5594
88.76	23.1-30	12	Single	7192
89.52	23.1-30	12	Dual	14384
92.56	30.5L-32	8	Single	9969
102.00	30.5L-32	8	Dual	19938

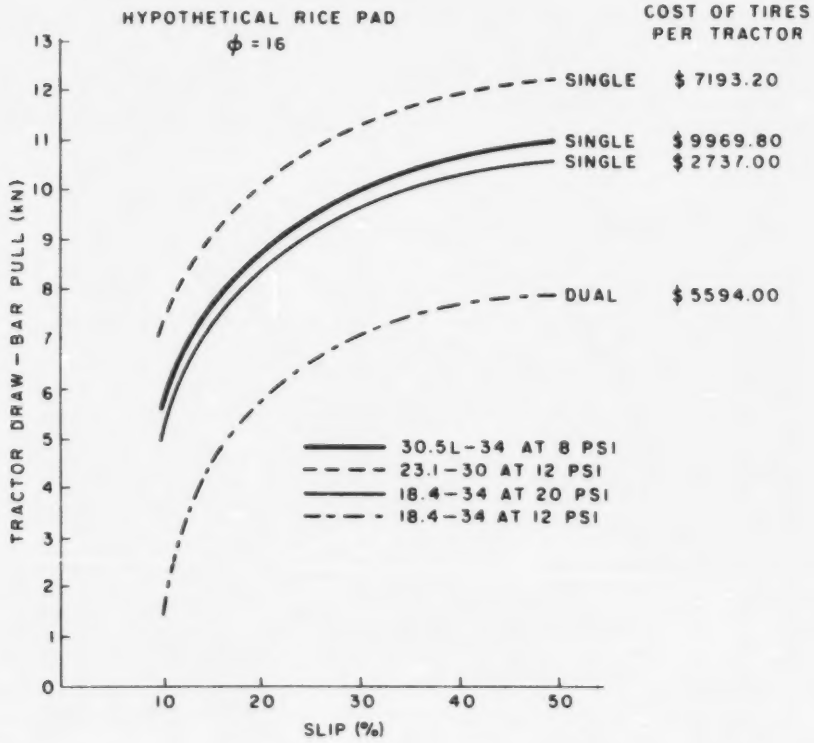

Figure 9. Drawbar pull of a 4×4 heavy tractor with a few tire options. Note again cost vs. performance gain. The parameters of a hypothetical rice pad were assumed on the basis of available data which may vary considerably. At lower friction angle, e.g., 16°, the tractor would be immobilized. Details in the text.

rice pad, for a complete tractor. Their performances are approximate as the second pass (rear wheel) soil properties were not available, and were assumed to be the same as for the first pass (front wheels). The assumption is tolerable if interpreted in terms of common experience (1). It is rather pessimistic because, in reality, an improvement of 10-25% of DP may usually be expected.

In both hypothetical cases of the clayey soil (Fig. 8) and rice pad (Fig. 9), the maximum tractor pulls reach 12-24 kN, at 30-50% slip, as opposed to 35-48% kN in other soils (Figs. 10 and 11). This is primarily due to the difference in the angle of soil, friction, ϕ = 25.8-27° (Figs. 10 and 11).

Soil cohesion plays a minimal role here because of the relatively small, short ground contact area. In cohesive soils, thrust may materialize only in substantially large contact areas, i.e., in tracks (6).

Tire performance in sand is subject to a paradox (Fig. 11): a single 18.4-34 tire at 20 psi inflation pressure (S20E) pulls 25% more than the same tire in dual configuration (D20R). The same is observed in 'woodland soil'. The situation dramatically changes when the inflation pressure is lowered to 12 psi (S12E, D12E). An explanation is offered in the next section. At this time, option selection may be quite baffling.

Snow layer 22 cm deep (Fig. 11) shows a disturbing lack of tire sensitivity to tire size and configuration, or inflation pressure, to accommodate better performances in pull-slip characteristics. Referring again to Fig. 10, note the paradoxical situation in turf, where 18.4-34 dual tire at 12 psi (D12E) is not much worse than a bigger, more expensive tire 30.5L-32 at 8 psi (S8E).

These examples graphically illustrate the direction in executing tire options, for various soils, from the viewpoint of traction, fuel-power consumption, weight, and cost. Many of the options considered in the old-fashioned way could be misleading because the result may not be apparent 'by eye'.

ANATOMY OF RELATIONSHIP BETWEEN TIRE-SOIL PARAMETERS

More insight in tire-soil relationships may be gained when analysing computation tapes which reveal what is not necessarily obvious in daily experience.

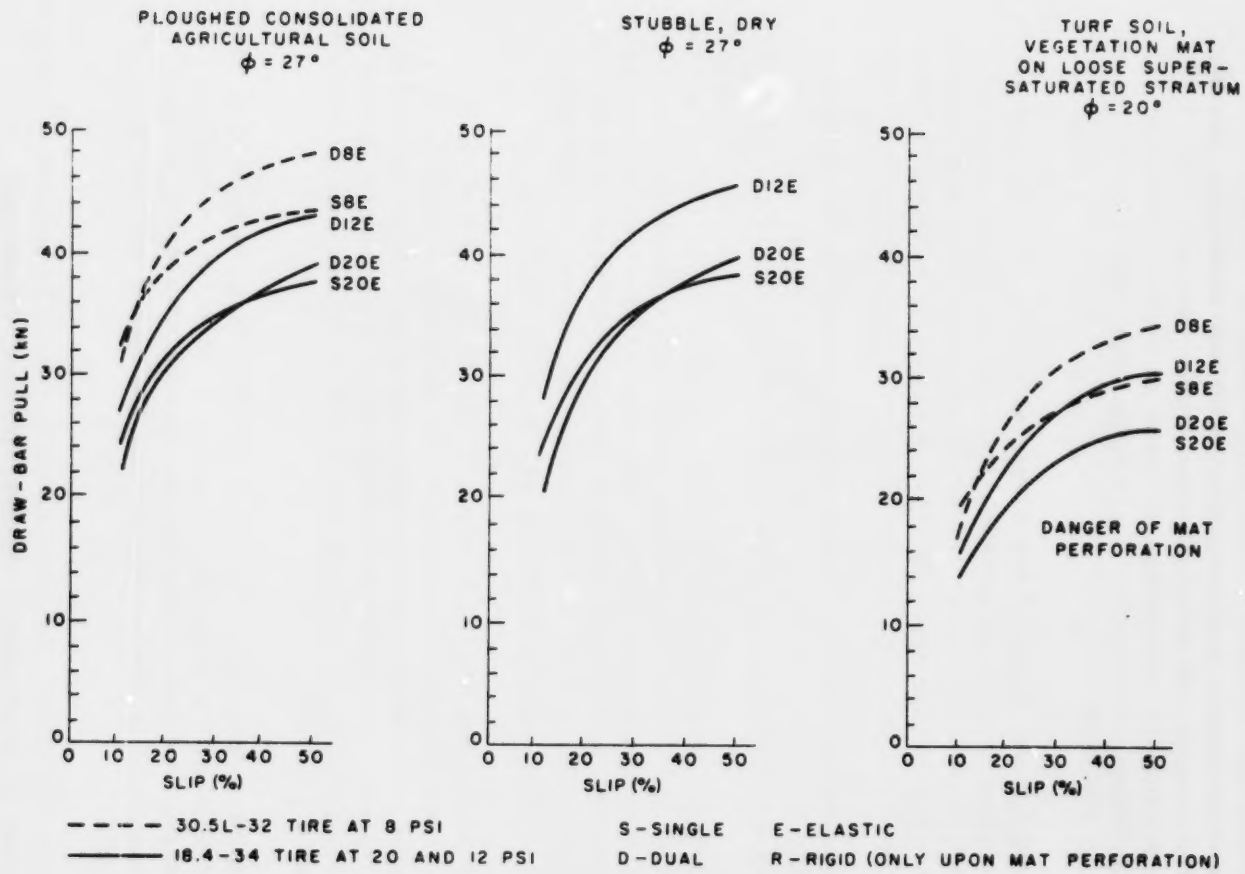

Figure 10. Drawbar pull of a heavy 4×4 tractor with available tire options in a number of soils. Note the poor performance of the tractor in turf, mainly due to low angle of friction, $\phi = 20°$. The overlapping performance of duals and singles is due to insignificant effect of the ground contact area in low-cohesion soils. Tire load is the main producer of traction.

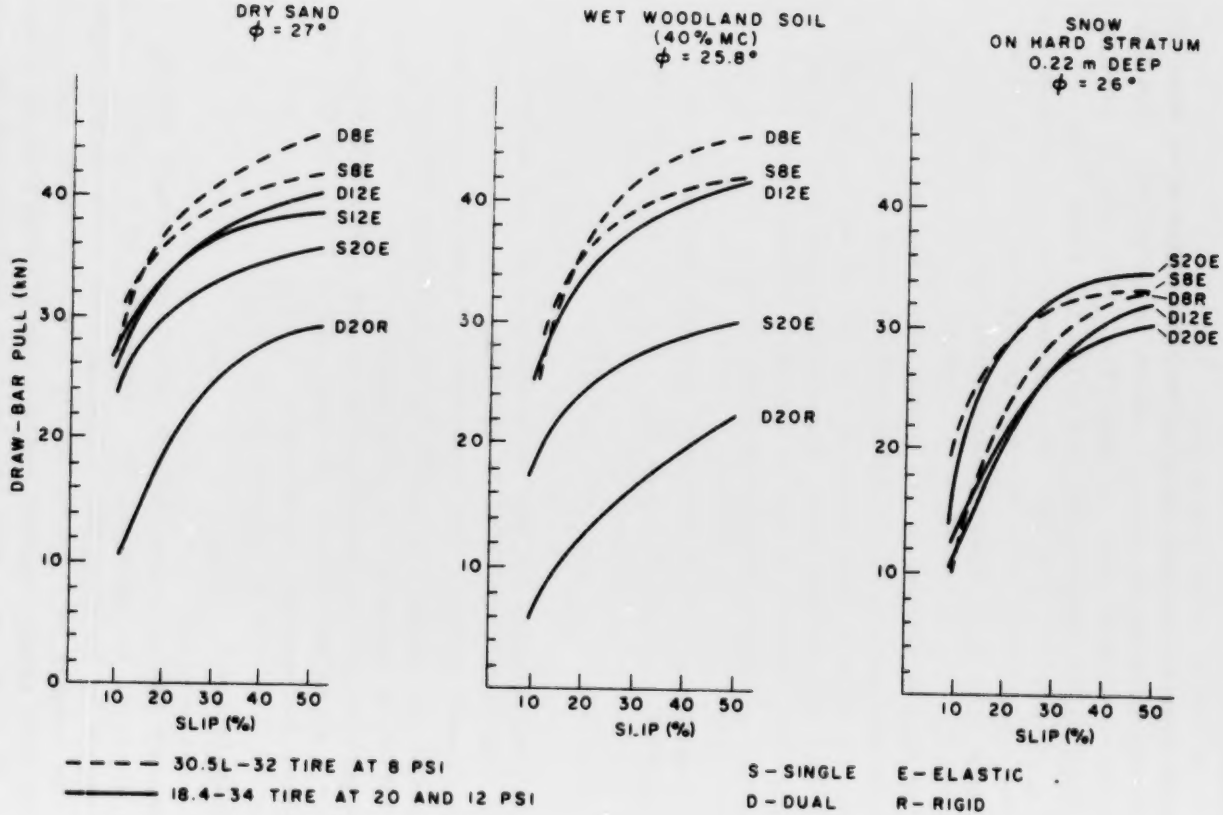

Figure 11. Drawbar pull of a heavy 4×4 tractor in various soils and snow. Note the paradox of the dual 18.4-34 tire with 20 psi inflation pressure which pulls less (D20R) than the same tire in a single configuration (S20E) because it had become rigid. Similar, often undetected, phenomenon may be observed in the "woodland soil". In snow, very little matters. Exercising tire options necessitates measurements and calculations.

Rigidity vs. Elasticity

Tire rigidity vs. elasticity, for example, may be observed on the background of parameters involved in Table 3, which is an extract from the HP-41C tape printed for a 18.4-34 tire, in sand (Fig. 11). The HP program compares z_e and z_r upon their computation, and calls for rigid tire processing because $z_r = 5.00 < z_e = 6.38$. Thus parameters b_{tr} and l_0 replace those for the elastic tire, b_0, l_1, l_2, in case of the D20R tire option. In other cases, the tires operate within the elastic TOM.

Tire rigidity is thus explained by the parameters involved. The abnormally high motion resistance, f = 0.1433, is explained by tire rigidity. The lesson is that a replacement of an 18.4-34 single tire at 20 psi, with a dual tire of the same size, necessitates the lowering of the pressure. In the case considered, 12 psi is enough. Calculations show that 15 psi also may be acceptable for loose sandy soil.

Note that the same situation was detected by computations in the 'woodland' soil (Fig. 11). It may not have been observed in the field.

Sinkage

Sinkage in duals and singles remains constant if load and inflation pressure remain the same. Only the size of the footprint changes, see Table 4. Also, the drawbar pull is nearly constant. However, the volume of soil displaced under the footprint caused by tire sinkage varies considerably with inflation pressure and tire configuration. The minimum coincides with the maximum of drawbar pull. Hence, the dual tire at 12 psi, producing DP = 40 kN at 4382 cm^3 of soil displacement, is the best option from efficiency and ecology viewpoints.

Ground Pressure Manipulation

The ground pressure manipulation through inflation pressure may be little effective. In sand, for example, the improvement of a single 18.4-34 tire performance by lowering the inflation from 20 to 12 psi, i.e., by 40%, increases the drawbar pull approximately 8.6%. Likewise, the configuration change from single to dual changes the drawbar pull by -17% at 20 psi and by +5.3% at 12 psi. This may be quite mind-boggling by conventional standards. The picture is more complex in cohesive soils, where the width of the ground contact area affects sinkage. In loose sandy soils considered here, the width is irrelevant.

Computations and experience with tires in snow (Fig. 11) or turf (Fig. 10) disclose similar surprises. Agricultural soils behave very much like sand, because of large angle of internal friction, ϕ.

Table 3. Behaviour of single and dual 18.4-34 tires at 20 and 12 psi and loads specified in Table 2

	Value (cm)			
	Inflation pressure = 20 psi		Inflation pressure = 12 psi	
Parameter	Dual	Single	Dual	Single
z_e	6.38	6.38	3.52	3.52
l_1	25.97	38.96	37.29	53.51
l_2	23.95	19.51	12.46	9.54
b_0	17.51	26.14	25.06	35.36
b_{tr}	40.00			
l_0	28.37			
δ	1.02	2.32	2.12	4.43
z_r	5.00	8.21	5.00	8.21
TOM	RIGID	Elastic	Elastic	Elastic
f_c	0.1330	0.0712	0.0442	0.0332
f_t		0.0092	0.0089	0.0212
f_b	0.0103	0.0089	0.0051	0.0027
f	0.1433	0.0892	0.0582	0.0571
Tire denotation	D20R	S20E	D12E	S12E

Table 4. Soil displacement and the drawbar pull of four options of 18.4-34 tire, in sandy soil

	Value (cm)			
	Inflation pressure = 20 psi		Inflation pressure = 12 psi	
Parameter	Dual	Single	Dual	Single
Print size (cm^2)	1135	1528	1245	2245
TOM	Rigid	Elastic	Elastic	Elastic
Sinkage (cm)	6.38	6.38	3.52	3.52
Displaced soil volume (cm^3)	7241	9749	4382	8008
Drawbar pull at 50% slip (kN)	29	35	40	38
Tire option	-	-	X	

Fuel Consumption

Fuel consumption depends on tire motion resistance (4). Reduction of inflation pressure may help. Table 3 shows that, when reducing inflation

from 20 to 12 psi, the coefficient of motion resistance f is reduced by the following values: singles, 0.0321; duals, 0.0541. Savings in energy consumption proportional to these figures may be expected. Note that dual configuration of the 18.4-34 tire is almost twice as sensitive to inflation pressure reduction as the single tire of the same size, in sand. In other soils, other sensitivity factors were found for the assumed soil properties, tire load and size.

This clearly suggests that a central inflation pressure control, in a vehicle, may not be precise if left to the subjective judgment of the driver. Torque-slip sensing devises would do a better job automatically if such a complexity and cost of equipment are justified.

Tread and Lugs

Tread and lugs are a common necessity. The question is, at what cost? Parametric analyses may throw some light, if not raise more questions.

Consider 10-28AS tire as computed in ploughed and stubble covered soil (Appendix, Tables A3 and A4). Computations were based on soil measurements and bevameter values reported in Figs. 1 and 2. Since only the lug tread provided the ground support as shown in Fig. 5, left, the lug induced motion resistance was computed accordingly. Table 5 shows an extract from calculator tapes. Pertinent parameters corresponding to tire performance in a ploughed soil, Fig. 5, right, are also recorded in Table 5.

Table 5 shows that in stubble the 1960 Model, assuming soil-filled inter-lug spaces, is inadequate because lug sinkage is smaller than lug height: $1.5376 < 2.0000$. This is why the model produces too low rolling resistance, $f = 0.0158$.

However, when using the lug calculations proposed in this paper, Appendix, Table A3, a correct solution is obtained: $f = 0.0697$, which is close to the value measured by Söhne and Sonnen, $f = 0.0640$ (8).

In the ploughed field, the inter-lug spaces are nearly occupied with trapped compacted soil ($z_L = 2.54 > 2.00$), hence the 1960 Model is closer to reality but not as close as the lug-cum-carcass computations show (Appendix, Table A4): $f = 0.0756$. This value better compares with the measured value $f = 0.0970$, as one may expect, considering test inadequacies (8).

The 1960 model is an integral part of lug-cum-carcass tire evaluation of the 1984 model. It represents a treadless torus with flexibility p_c

Table 5. 10-28AS tire motion resistance in a stubble and ploughed field. Verification of tire model selection

Parameter	Value			
	Stubble soil, LBV		Ploughed soil, LBV	
	Carcass support, no lugs action	Lug support, chevron only	Carcass support, no lugs action	Lug support, chevron only
z_e	0.0267		1.8390	
z_r	0.8627	0.8627	4.0686	4.0686
z_c	0.0267		1.8390	0.5400
z_L		1.5376		2.5400
h_L	2.0	2.0	2.0	2.0
TOM	Elastic	Elastic	Elastic	Elastic
f_c	0.0007	0.0384	0.0324	0.0379
f_t	0.0148	0.0148	0.0114	0.0114
f_b	0.0003	0.00165	0.0202	0.0263
f	0.0158	0.0697	0.0640	0.0756
Measured		0.0640		0.0970
Calc. model	1960	1984	1960	1984

that includes tread. Thus, the difference between 1960 and 1984 models reflects the drag induced by the lugs, or tread.

For tire load 700 kg, the additional motion resistance R caused by the lugs is, in the stubble soil, quite sizeable:

$$R = 700(0.0697 - 0.0158) = 37.73 \text{ kg}$$

per one tire, or 75.46 per tractor. In the ploughed soil, the lugs are not detrimental. Considering the difference, 0.0640-0.0756 as enclosed within the calculation, or experimental error, or both, one may say that the lugs are irrelevant. Both tire models produce the same result, in agreement with practice.

The question as to why and when to introduce lugs of specific form and size emerges clearly. The problem deserves some experimental and parametric analyses. Experiments alone may be misleading.

CLOSING REFLECTIONS

In this paper, I tried to update and possibly finalize my concept of the engineering mechanics of off-road locomotion, based on a unified soil

value system and practical methodology. Although the paper refers to pneumatic tires only, the reader familiar with my work may recognize that the examples and illustrations discussed have a broader meaning applicable to all kinds of roadless locomotion.

It is hoped that this presentation may help: first, the problem formulation, from an engineering viewpoint; second, the selection of a method which would enable the manufacturer, user, and the designer to perform parametric analyses, without which their respective tasks have always been beyond the limits of full accomplishment. In a final observation, let me note that the progress in the area discussed has been peripheral and evolved along the geometrical lines of heavier and larger equipment. No radical departure from the Form-Size-Power package, which has existed for decades, has been observed.

Parametric analyses offer a fascinating, computerized game of playing with unorthodox ideas. Perhaps there lies the future.

ACKNOWLEDGEMENTS

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APPENDIX A: GLOSSARY

Equations	HP 41C Program	Symbol and Denotation Description
A/C	A/C	Area of tire print
A/L	A/L	Area of lug print
A/CN	A/CN	Net area of tire print, A/C - A/L
b_0	BO BSUBO	Width of the ground contact area
b_c	B/C	Width of average space between lugs
b_L	B/L	Width of the lug print
b_{tr}	BTR	Tire tread width
c	C	soil cohesion
D	D	Tire diameter
h	H	Tire carcass height
h_1	H/L	Height of the lug
H	T	Soil thrust
H-R	T-R	Draw-bar pull
i_0	I	Tire slip
k_c	Kc	Cohesive coefficient of soil deformation
k_ϕ	KPHI	Frictional coefficient of soil deformation
K	K	Coefficient of soil slip
K_c	K_c	Terzaghian parameters of soil
K_γ	KGAMMA	Terzaghian parameters of soil
l_0	LSUBR	Length of the rigid tire ground contact
l_1	LSUB1	Length of the flat portion of tire print
l_2	LSUB2	Length of the curved portion of the tire
n	N	Exponent of soil deformation
p_g	PG	Uniform tire ground pressure
R		Total motion resistance
R_b	Fb	Motion resistance, bulldozing
R_c	Fc	Motion resistance, compaction
R_{cC}	RC/C	Carcass compaction resistance
R_{cL}	RC/L	Lug compaction resistance
$R_{f,r}$	R/FAX,RAX	Front axle motion resistance, rear axle motion resistance
R_t	FT	Motion resistance, carcass flexing
W	W	Tire load
z_c	z	Tire carcass sinkage
z_e	ZSUBe	Sinkage of elastic tires
z_r	ZSUBR	Sinkage of rigid tires
z_L	Z/L	Lug sinkage
2r	2R	Double radius of tire tread

Equations	HP 41C Program	Symbol and Denotation Description
γ	GAMMA	soil density
δ	DELTA	Tire deflection
λ	ETA1	Tire print "ellipticity" coefficient
η_1	ETA1	The same as λ
η_2	ETA2	Ratio of lug to carcass ground contact area areas
ϕ	PHI	Angle of soil friction

Table A1. Computation, 11.00-20 butted tire

Motion Resistance

Ground Contact Area Parameters

Additional Soil Input and Registers

```
11.00-20 BUFFED TIRE

DRY SAND
                    XEQ "ENT"
KPHI= R2
Kc  = R3
H   = R4
W   = R5
D   = R6
BTR = R7
ZR  = R8
H   = R9
PG  = R10
ENGLISH MEASURES
```

Register Denotations for Main Input Parameters

Main Input Parameters

```
          5.0000 STO 02
          1.5000 STO 03
          0.0000 STO 04
      3,000.0000 STO 05
         42.7000 STO 06
         11.0000 STO 07
         22.0000 STO 08
         10.0000 STO 09
         17.0000 STO 10
              XEQ "ST"
```

Checking for TOM

```
B0=?
R/S TO BEGIN
          9.5697    RUN
          9.5698    ***
          9.5696    ***
          9.5698    ***
          9.5697    ***

B SUB 0=9.5697
Z SUB e=4.4421
L SUB 2=0.8920
L SUB 1=13.5005
DELTA  =1.0952
BTR    =11.0000

STOH

L SUB R=15.4732
BTR    =11.0000

Z SUB R=6.6394
USE Ze

INPUT
C     R14
GAMMA R15
XGAMMA R23
Kc    R24
          0.0000 STO 14
          0.0600 STO 15
         20.0000 STO 23
          0.0000 STO 24
                    RUN
Fc=0.1338
FT=0.0166
Fb=0.0434
F =0.1938
```

Iteration for b_o

Additional Soil Parameters

Thrust and Draw-bar Pull

```
THRUST INPUT

PHI R13
K   R19
          34.0000 STO 13
           0.0200 STO 19
                    RUN
I =0.1000
T =2.005.4523
T-R =1.423.9059

I =0.2000
T =2.014.4089
T-R =1.433.0225

I =0.3000
T =2.017.5011
T-R =1.436.0340

I =0.4000
T =2.019.0072
T-R =1.437.5409

I =0.5000
T =2.019.9109
T-R =1.438.4445
```

Table A2. Computation, 5.50-16 tire, stubble, ploughed field

```
5.50-16  STUBBLE  LBV
          3.7000 STO 02
          3.0000 STO 03
          0.3400 STO 04
        245.0000 STO 05
         70.0000 STO 06
         13.0000 STO 07
         20.0000 STO 08
         11.5000 STO 09
          1.8500 STO 10
              XEQ "ST"

BO=?
R/S TO BEGIN
          8.2609    RUN
          8.2608    ***
          8.2609    ***
          8.2603    ***

B SUB 0=8.2608
Z SUB e=0.0909
L SUB 2=0.4300
L SUB 1=15.7104
DELTA  =0.0929
BTR    =13.0000

STOW

L SUB P=6.5921
BTR    =13.0000

Z SUB P=0.5062
USE Ze

INPUT

C     R14
GAMMA R15
KGAMMA R23
Kc    R24
          0.0400 STO 14
          1.0000    CLX
          0.0012 STO 15
         15.0000 STO 23
         13.0000 STO 24
                    RUN

Fc=0.0046
FT=0.0130
Fb=0.0018
F =0.0194

A/L=42.4178
Z/L=1.0397

Fc=0.0494
Fb=0.0124
FT=0.0130

F=0.0748

P/FAX=36.6520
```

```
5.50-16  PLOUGHED  LBV
              XEQ "ENT"

KPHI= R2
Fc  = R3
W   = R4
W   = R5
D   = R6
BTR = R7
2P  = R8
H   = R9
PG  = R10
          0.5000 STO 02
          4.0000 STO 03
          0.7000 STO 04
        245.0000 STO 05
         70.0000 STO 06
         13.0000 STO 07
         20.0000 STO 08
         11.5000 STO 09
          1.8500 STO 10
              XEQ "ST"

BO=?
R/S TO BEGIN
          7.3327    RUN
          7.3328    ***
          7.3327    ***
          7.3327    ***
          7.3327    ***

B SUB 0=7.3327
Z SUB e=2.0785
L SUB 2=7.4165
L SUB 1=13.8939
DELTA  =0.6964
BTR    =13.0000

STOW

L SUB P=13.8659
BTR    =13.0000

Z SUB P=2.0638
USE Ze
```

```
INPUT

C     R14
GAMMA R15
KGAMMA R23
Kc    R24
          0.0400 STO 14
          0.0012 STO 15
         15.0000 STO 23
         13.0000 STO 24
                    RUN

Fc=0.0647
FT=0.0077
Fb=0.0403
F =0.1127

INPUT

ETA=0.8
B/L=1.5
H/L=1.3

B/C=1.4442
A/C=81.5038
A/L=37.5135
Z/L=2.52% > 1.3
A/CN=43.9903

245=143.0351 Z EXP 0.78+
118.7928/Z+1.3/EXP 0.78

Z=0.375
RC/L=12.0307
RC/CN=1.0148

Fc=0.0532
Fb=0.0181
FT=0.0077
F =0.0790

P/FAX=38.7100
```

Table A3. Computation, 10-28AS tire, stubble

```
10--28 AS  STUBBLE  LBV

          3.7000 STO 02
          3.0000 STO 03
          0.3400 STO 04
        727.0000 STO 05
        122.0000 STO 06
         22.0000 STO 07
         65.0000 STO 08
         23.0000 STO 09
          1.1200 STO 10
              XEQ "ST"

BO=?
R/S TO BEGIN
         21.6635    RUN
         21.6632    ***
         21.6635    ***
         21.6633    ***


B SUB 0=21.6633
Z SUB e=0.0267
L SUB 2=0.1007
L SUB 1=29.8823
DELTA  =1.3581
BTR    =22.0000

STAN


L SUB R=10.4588
BTR    =22.0000

Z SUB R=0.8627
USE Ze
INPUT

C     R14
GAMMA R15
KGAMMA R23
Kc    R24
```

```
          0.0400 STO 14
          0.0012 STO 15
         15.0000 STO 23
         13.0000 STO 24
                    RUN
Fc=0.0007
FT=0.0148
Fb=0.0003
F =0.0157

THRUST INPUT

PHI R13
K   R19
         20.0000 STO 13
          2.0000 STO 19
                    RUN
I =0.1000
T =199.0306
T-R =187.5929

I =0.2000
T =282.1165
T-R =270.6708

I =0.3000
T =322.1728
T-R =310.7271

I =0.4000
T =344.2709
T-R =332.8252

I =0.5000
T =357.8995
T-R =346.4538
```

```
INPUT

ETA 1=0.9
ETA 2=0.22
B/L=2.5
N/L=2

A/L=128.1763
Z/L=1.5376

Fc=0.0304
Fb=0.0165
FT=0.0148
F =0.0697

R/RAX=101.3438
```

Table A4. Computation, 10-28AS tire, ploughed field

```
10-28 AS  PLOUGHED  LOV
          0.5000 STO 02
          4.0000 STO 03
          0.7000 STO 04
        727.0000 STO 05
        122.0000 STO 06
         22.0000 STO 07
         65.0000 STO 08
         23.0000 STO 09
          1.1200 STO 16
             XEQ "ST"

BO=?
R/S TO BEGIN
          0.0000    CLX
         20.3706    RUN
         20.3703    ***
         20.3707    ***
         20.3704    ***
         20.3705    ***


B SUB 0=20.3705
Z SUB e=1.8390
I SUB 2=6.7452
I SUB 1=20.0756
DELTA  =1.6372
BTR    =22.0000

STOW


I SUB R-21.9135
BTR    =22.0000

Z SUB R=4.0686
USE Ze

INPUT

C     R14
GAMMA R15
KGAMMA R23
Kc    R24
```

```
          0.0400 STO 14
          0.0012 STO 15
         15.0000 STO 23
         13.0000 STO 24
                 RUN
Fc=0.0324
FT=0.0114
Fb=0.0202
F =0.0641

THRUST INPUT

PHI R13
K   R19
         24.0000 STO 13
          1.5000 STO 19
                 RUN
I =0.1000
T =216.6685
T-R =170.1022

I =0.2000
T =278.7416
T-R =232.1753

I =0.3000
T =303.4937
T-R =256.9274

I =0.4000
T =316.1686
T-R =269.6023

I =0.5000
T =323.7971
T-R =277.2300


INPUT
ETA 1=0.9
ETA 2=0.22
B/L=2.5
H/L=2
B/C=9
```

```
B/C=514.7215
B/L=113.2387
Z/L=4.1899 > 2
B/CH=401.4828

727=379.1702 Z EXP 0.78+
237.0013/Z+2/EXP 0.78

Z=0.54
BC/L=25.0076
BC/CH=2.5337

Fc=0.0379
Fb=0.0263
FT=0.0114
F =0.0756

R/MAX=189.9224
```